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(NS5, Dp) and (NS5, D$(p+2)$, Dp) bound states of type-IIB and type-IIA string theories

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Abstract: Starting from the (q, p) 5-brane solution of type-IIB string theory, we here construct the low energy configuration corresponding to (NS5, Dp)-brane bound states (for $0 \leq p \leq 4$) using the T-duality map between type-IIB and type-IIA string theories. We use the SL$(2, \mathbb{Z})$ symmetry on the type-IIB bound state (NS5, D3) to construct (NS5, D5, D3) bound state. We then apply T-duality transformation again on this state to construct the bound states of the form (NS5, D$(p+2)$, Dp) (for $0 \leq p \leq 2$) of both type-IIB and type-IIA string theories. We give the tension formula for these states and show that they form non-threshold bound states. All these states preserve half of the space-time supersymmetries of string theories. We also briefly discuss the ODp-limits corresponding to (NS5, Dp) bound state solutions.

Keywords: Superstrings and Heterotic Strings, p-branes, D-branes.

Contents

1. Introduction

The low energy effective actions of type II string theories are known to possess an NS5-brane solution [1]–[5] which is the magnetic dual to the fundamental string solution [6, 7]. This solution is non-singular and purely solitonic and therefore, not much is known about its dynamics. Type II string theories also contain Dp-branes [8, 9] in their low energy spectrum and it is well-known that Dp-branes can end on type-IIA NS5-branes for $p=$ even and they can end on type-IIB NS5-branes for $p=$ odd. It is therefore expected that these Dp-branes will form bound states with NS5-branes. The bound state (NS5, D5) of type-IIB theory known as $(q,\ p)$ fivebranes, with q, p relatively prime integers corresponding to the charges of NS5-branes and D5-branes respectively has already been constructed in ref. [10]. We here use this solution and apply the T-duality map from type-IIB to type-IIA theory also from type IIA to type-IIB theory to construct the (NS5, Dp) bound state solutions for $0 \leq p \leq 4$. The T-duality is applied along the longitudinal directions of D5-branes. We note that the (NS5, Dp) bound states have also been given in [11, 12] and they were obtained from (NS5, D1) solution (constructed by applying S-duality on already known [13] (F, D5) solution) and applying T-duality along the transverse directions of D-string. However, the (NS5, D5) solution obtained in this way does not agree with the already known solution. We have indicated the possible reason for this discrepancy in section 2. This is the reason we have chosen to start from the known (NS5, D5) solution to construct the (NS5, Dp) solutions.

In order to construct (NS5, D$(p+2)$, Dp) bound states for $0 \leq p \leq 3$, we start from (NS5, D3) solution of type-IIB string theory. Since type-IIB string theory is conjectured to have a non-perturbative quantum $\mathrm{SL}(2,\mathbb{Z})$ symmetry, we use this symmetry to construct (NS5, D5, D3) bound state first. Then we apply T-duality map from type-IIB (IIA) to type-IIA (IIB) theory along the longitudinal directions of D3-branes to construct (NS5, D$(p+2)$, Dp) bound states. We derive the tension formula for both (NS5, Dp) and (NS5, D$(p+2)$, Dp) bound state solutions and show that they form non-threshold bound states. For (NS5, Dp) case they are characterized by two relatively prime integers and for (NS5, D$(p+2)$, Dp) case they are characterized by three integers where any two of them are relatively prime. Since S- and T-duality do not break supersymmetry of the system, all these states preserve half of the space-time supersymmetries as the original (NS5, D5) solution of type-IIB string theory.

Apart from being interesting on their own, one of the motivations for studying the bound states of type II NS5-branes with Dp-branes is to look at the world-volume theory of NS5-branes in the presence of various Dp-branes. It is well-known that the world-volume theory of pure NS5-branes in the decoupling limit does not produce a local field theory. But gives a non-gravitational, non-local theory known as the little string theory in (5+1)-dimensions [14]–[19]. It is therefore, natural to look at the corresponding theory in the presence of various Dp-branes since they form bound states with NS5-branes.[1]. It is pointed out in ref. [21] that NS5-branes in the presence of a critical RR electric field (Dp-branes) reduce to again a non-gravitational and non-local theory (different from little string theory) known as the (5+1)-dimensional light open Dp-brane (ODp) theories in a particular decoupling limit. The (NS5, Dp) bound state solutions in the decoupling limit give the supergravity dual of ODp-theories. These supergravity solutions of ODp theories are also discussed in ref. [11]. Since our solutions differ from those given in [11], we also briefly discuss the ODp-limits for the solutions constructed in this paper. We have not studied the decoupling limits for (NS5, D$(p+2)$, Dp) bound states in this paper, but it would be interesting to investigate what kind theories do they correspond to and will be reported elsewhere.

The paper is organized as follows. In section 2, we give the construction of (NS5, Dp) bound state solution starting from $(q,\, p)$ 5-branes by applying T-duality map along the longitudinal directions of D5-branes. We also give the tension formula for these bound states. In section 3, starting from the type-IIB bound state (NS5, D3), we construct the (NS5, D5, D3) bound state solution by applying $SL(2,\mathbb{Z})$ symmetry of type-IIB theory. In section 4, we construct the (NS5, D$(p+2)$, Dp)-brane bound states by applying T-duality on (NS5, D5, D3) along the longitudinal directions of D3-branes. Here also we give the corresponding tension formula. Finally, in section 5, we briefly discuss the ODp limits for the (NS5, Dp) solutions obtained in section 2.

[1]Supergravity solutions involving NS5-branes in the presence of RR fields have also been discussed in [20].

2. (NS5, Dp) bound states

Type IIB string theory is well-known to possess an $SL(2,\mathbb{Z})$ multiplet of magnetically charged 5-brane solution known as (q,p) 5-branes first constructed in ref.[10]. In this section we will start with this solution and apply T-duality map between type-IIB and type-IIA string theories along the longitudinal directions of 5-branes to construct the (NS5, Dp) bound state solutions. The (q,p) 5-brane solution, with (q,p) relatively prime integers, denoting the charges of NS5-brane and D5-brane respectively is given as,

(NS5, D5) solution.

$$\begin{aligned}
ds^2 &= H^{1/2}H'^{1/2}\left[H^{-1}\left(-dx_0^2+dx_1^2+\cdots+dx_5^2\right)+dr^2+r^2d\Omega_3^2\right]\\
e^{\phi_b} &= g_s H' H^{-1/2}\\
B^{(b)} &= 2Q_5\cos\varphi\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2\\
A^{(2)} &= -\frac{2Q_5\sin\varphi}{g_s}\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2\\
\chi &= -\frac{p}{q}(1-H'^{-1}), \qquad\qquad A^{(4)}=0\,,
\end{aligned}\tag{2.1}$$

where in the above, we have written the metric in the string-frame. Also, $d\Omega_3^2 = d\theta^2+\sin^2\theta d\phi_1^2+\sin^2\theta\sin^2\phi_1 d\phi_2^2$ is the line element for the unit 3-sphere transverse to the 5-branes, $g_s=e^{\phi_0}$ is the asymptotic value of the dilaton, $B^{(b)}$ and $A^{(2)}$ denote the NSNS and RR two-form potentials. χ is the RR scalar and $A^{(4)}$ is the RR 4-form gauge field. The harmonic functions H and H' are given as,

$$\begin{aligned}
H &= 1+\frac{Q_5}{r^2}\\
H' &= 1+\frac{\cos^2\varphi Q_5}{r^2}\,,
\end{aligned}\tag{2.2}$$

where the angle $\cos\varphi = q/(\sqrt{p^2g_s^2+q^2})$ and Q_5 is defined as,

$$Q_5 = \sqrt{p^2g_s^2+q^2}\,\alpha' \tag{2.3}$$

Note that the solution given here is different from the one constructed in [11, 12]. The axion in this case vanishes asymptotically, whereas it is constant in [12]. Also, the RR 3-form field strength is constant here, but in [12] it is proportional to χ. The other fields are the same.

The T-duality map from type-IIB fields to type-IIA fields are given as [22],

$$\begin{aligned}
G_{\tilde{x}\tilde{x}} &= \frac{1}{J_{\tilde{x}\tilde{x}}}\,, \qquad G_{\tilde{x}\mu} = -\frac{B^{(b)}_{\tilde{x}\mu}}{J_{\tilde{x}\tilde{x}}}\\
G_{\mu\nu} &= J_{\mu\nu} - \frac{J_{\tilde{x}\mu}J_{\tilde{x}\nu} - B^{(b)}_{\tilde{x}\mu}B^{(b)}_{\tilde{x}\nu}}{J_{\tilde{x}\tilde{x}}}
\end{aligned}$$

$$
\begin{aligned}
e^{2\phi_a} &= \frac{e^{2\phi_b}}{J_{\tilde{x}\tilde{x}}} \\
B^{(a)}_{\tilde{x}\mu} &= -\frac{J_{\tilde{x}\mu}}{J_{\tilde{x}\tilde{x}}}\,, \qquad B^{(a)}_{\mu\nu} = B^{(b)}_{\mu\nu} + 2\frac{B^{(b)}_{\tilde{x}[\mu}J_{\nu]\tilde{x}}}{J_{\tilde{x}\tilde{x}}} \\
A^{(1)}_{\tilde{x}} &= -\chi\,, \qquad A^{(1)}_{\mu} = A^{(2)}_{\tilde{x}\mu} + \chi B^{(b)}_{\tilde{x}\mu} \\
A^{(3)}_{\tilde{x}\mu\nu} &= A^{(2)}_{\mu\nu} + 2\frac{A^{(2)}_{\tilde{x}[\mu}J_{\nu]\tilde{x}}}{J_{\tilde{x}\tilde{x}}} \\
A^{(3)}_{\mu\nu\rho} &= A^{(4)}_{\mu\nu\rho\tilde{x}} + \frac{3}{2}\left(A^{(2)}_{\tilde{x}[\mu}B^{(b)}_{\nu\rho]} - B^{(b)}_{\tilde{x}[\mu}A^{(2)}_{\nu\rho]} - 4\frac{B^{(b)}_{\tilde{x}[\mu}A^{(2)}_{\tilde{x}\nu}J_{\rho]\tilde{x}}}{J_{\tilde{x}\tilde{x}}}\right).
\end{aligned}
\tag{2.4}
$$

Here $\tilde{x}$ is the Killing coordinate along which T-duality is performed. μ, ν, ρ denote any coordinate other than $\tilde{x}$. J and G are the string-frame metric in type-IIB and type-IIA string theory respectively. ϕ_a, ϕ_b are the dilatons and $B^{(b)}$, $B^{(a)}$ are the NSNS two-form gauge fields in these two theories. $A^{(1)}$ and $A^{(3)}$ are the RR 1-form and 3-form gauge fields in type-IIA theory whereas, χ, $A^{(2)}$ and $A^{(4)}$ are the RR scalar, 2-form and 4-form gauge fields in type-IIB string theory. The field strengths which appear in the corresponding low energy actions are given as,[2]

$$
\begin{aligned}
H^{(a)} &= dB^{(a)} \\
H^{(b)} &= dB^{(b)} \\
F^{(2)} &= dA^{(1)} \\
F^{(3)} &= dA^{(2)} \\
F^{(4)} &= dA^{(3)} - H^{(a)} \wedge A^{(1)} \\
F^{(5)} &= dA^{(4)} - \frac{1}{2}\left(B^{(b)} \wedge F^{(3)} - A^{(2)} \wedge H^{(b)}\right).
\end{aligned}
\tag{2.5}
$$

Note that the RR 5-form field strength $F^{(5)}$ in type-IIB theory is self-dual i.e. $F^{(5)} = *F^{(5)}$, with $*$ denoting the Hodge-dual and we will use this fact to find out the gauge field $A^{(4)}$ in (NS5, D3) solution in the following.

Now a straightforward application of the T-duality rule given in eq. (2.4) on the type-IIB background (1) along x^5 coordinate yields the (NS5, D4) bound state configuration in type-IIA theory as,

(NS5, D4) solution.

$$
\begin{aligned}
ds^2 &= H^{1/2}H'^{1/2}\left[H^{-1}\left(-dx_0^2 + dx_1^2 + \cdots + dx_4^2\right) + H'^{-1}dx_5^2 + dr^2 + r^2 d\Omega_3^2\right] \\
e^{\phi_a} &= g_s H'^{3/4} H^{-1/4} \\
B^{(a)} &= 2Q_5 \cos\varphi \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2
\end{aligned}
$$

[2]In the literature, the 3-form field-strength $F^{(3)}$ (defined below) is sometimes redefined to include a Chern-Simons term as $F'^{(3)} = F^{(3)} + \chi H^{(b)}$, such that the kinetic energy term takes the usual form. But, since this is not important for our purpose, we will keep the definition as given in eq. (2.5).

$$
\begin{aligned}
A^{(1)} &= \frac{p}{q}(1 - H'^{-1})dx^5 \\
A^{(3)} &= -\frac{2Q_5 \sin\varphi}{g_s} \sin^2\theta \cos\phi_1 dx^5 \wedge d\theta \wedge d\phi_2 \,. \qquad (2.6)
\end{aligned}
$$

Now in order to get (NS5, D3) solution from here, we have to apply T-duality transformation along x^4-direction. Since the solution (6) belongs to type-IIA theory, we have to use the T-duality map from type-IIA to type-IIB fields. They are given as follows [22],

$$
\begin{aligned}
J_{\tilde{x}\tilde{x}} &= \frac{1}{G_{\tilde{x}\tilde{x}}}\,, \qquad J_{\tilde{x}\mu} = -\frac{B^{(a)}_{\tilde{x}\mu}}{G_{\tilde{x}\tilde{x}}} \\
J_{\mu\nu} &= G_{\mu\nu} - \frac{G_{\tilde{x}\mu}G_{\tilde{x}\nu} - B^{(a)}_{\tilde{x}\mu}B^{(a)}_{\tilde{x}\nu}}{G_{\tilde{x}\tilde{x}}} \\
e^{2\phi_b} &= \frac{e^{2\phi_a}}{G_{\tilde{x}\tilde{x}}} \\
B^{(b)}_{\tilde{x}\mu} &= -\frac{G_{\tilde{x}\mu}}{G_{\tilde{x}\tilde{x}}}, \qquad B^{(b)}_{\mu\nu} = B^{(a)}_{\mu\nu} + 2\frac{G_{\tilde{x}[\mu}B^{(a)}_{\nu]\tilde{x}}}{G_{\tilde{x}\tilde{x}}} \\
\chi &= -A^{(1)}_{\tilde{x}}, \qquad A^{(2)}_{\tilde{x}\mu} = A^{(1)}_{\mu} - \frac{A^{(1)}_{\tilde{x}}G_{\tilde{x}\mu}}{G_{\tilde{x}\tilde{x}}} \\
A^{(2)}_{\mu\nu} &= A^{(3)}_{\mu\nu\tilde{x}} - 2A^{(1)}_{[\mu}B^{(a)}_{\nu]\tilde{x}} + 2\frac{G_{\tilde{x}[\mu}B^{(a)}_{\nu]\tilde{x}}A^{(1)}_{\tilde{x}}}{G_{\tilde{x}\tilde{x}}} \\
A^{(4)}_{\mu\nu\rho\tilde{x}} &= A^{(3)}_{\mu\nu\rho} - \frac{3}{2}\left(A^{(1)}_{[\mu}B^{(a)}_{\nu\rho]} - \frac{G_{\tilde{x}[\mu}B^{(a)}_{\nu\rho]}A^{(1)}_{\tilde{x}}}{G_{\tilde{x}\tilde{x}}} + \frac{G_{\tilde{x}[\mu}A^{(3)}_{\nu\rho]\tilde{x}}}{G_{\tilde{x}\tilde{x}}}\right). \qquad (2.7)
\end{aligned}
$$

Here as before, $\tilde{x}$ is the Killing coordinate along which T-duality transformation is performed and μ, ν, ρ are any coordinate other than $\tilde{x}$. Also, the remaining components of $A^{(4)}$ can be obtained from the self-duality condition on the corresponding 5-form field-strength. Thus applying T-duality transformation along x^4-coordinate we obtain (NS5, D3) solution of type-IIB theory as follows:

(NS5, D3) solution.

$$
\begin{aligned}
ds^2 &= H^{1/2}H'^{1/2}\left[H^{-1}\left(-dx_0^2 + dx_1^2 + \cdots + dx_3^2\right) + H'^{-1}\left(dx_4^2 + dx_5^2\right) + dr^2 + r^2 d\Omega_3^2\right] \\
e^{\phi_b} &= g_s H'^{1/2} \\
B^{(b)} &= 2Q_5 \cos\varphi \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2 \\
\chi &= 0 \\
A^{(2)} &= \frac{p}{q}(1 - H'^{-1})dx^4 \wedge dx^5 \\
A^{(4)} &= -\frac{Q_5 \sin\varphi}{g_s}(1 + H'^{-1}) \sin^2\theta \cos\phi_1 dx^4 \wedge dx^5 \wedge d\theta \wedge d\phi_2 - \\
&\quad -\frac{\sin\varphi}{g_s} H^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,. \qquad (2.8)
\end{aligned}
$$

We would like to point out that the T-duality rule given in (2.7) produces only the first term of $A^{(4)}$ in eq. (2.8). However, since D3-branes are self-dual, we have to include the term obtained by applying self-duality on the corresponding field strength. This is how we obtained the second term of $A^{(4)}$ above.[3] This is also important to obtain the correct form of (NS5, D4) and (NS5, D5) solution if we apply T-duality in the transverse directions of D3-branes in (NS5, D3) solution. We think this is the reason why (NS5, D5) solution obtained in [12] differs from eq. (2.1).

We now apply T-duality map from type-IIB fields to type-IIA fields given in eq. (2.4) along x^3-coordinate on (NS5, D3) solution to obtain (NS5, D2) bound state configuration,

(NS5, D2) solution.

$$\begin{aligned}
ds^2 &= H^{1/2}H'^{1/2}\left[H^{-1}(-dx_0^2+dx_1^2+dx_2^2)+H'^{-1}(dx_3^2+dx_4^2+dx_5^2)+dr^2+r^2d\Omega_3^2\right]\\
e^{\phi_a} &= g_s H^{1/4}H'^{1/4}\\
B^{(a)} &= 2Q_5\cos\varphi\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2\\
A^{(1)} &= 0\\
A^{(3)} &= \frac{p}{q}(1-H'^{-1})dx^3\wedge dx^4\wedge dx^5-\frac{\sin\varphi}{g_s}H^{-1}dx^0\wedge dx^1\wedge dx^2\,.
\end{aligned} \tag{2.9}$$

This state belongs to type-IIA theory. So applying T-duality map from type-IIA to type-IIB fields given in (7) along x^2-coordinate, we obtain (NS5, D1) bound state as follows:

(NS5, D1) solution.

$$\begin{aligned}
ds^2 &= H^{1/2}H'^{1/2}\left[H^{-1}\left(-dx_0^2+dx_1^2\right)+H'^{-1}\left(dx_2^2+\cdots+dx_5^2\right)+dr^2+r^2d\Omega_3^2\right]\\
e^{\phi_b} &= g_s H^{1/2}\\
B^{(b)} &= 2Q_5\cos\varphi\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2\\
\chi &= 0\\
A^{(2)} &= -\frac{\sin\varphi}{g_s}H^{-1}dx^0\wedge dx^1\\
A^{(4)} &= -\frac{p}{q}(1-H'^{-1})dx^2\wedge dx^3\wedge dx^4\wedge dx^5\,.
\end{aligned} \tag{2.10}$$

Finally the (NS5, D0) bound state can be obtained from the above type-IIB bound state configuration by applying T-duality map eq. (2.4) along x^1-direction. The solution is:

[3] Note here that the Hodge duality is taken with respect to the string-frame metric given in eq. (2.8). However, since the dilaton is not constant in this case one might think that we will get different answer if the Einstein-frame metric $ds_E^2 = e^{-\phi_b/2}ds^2$ is used instead. But, it can be easily checked that the dilaton factor cancels while calculating the Hodge dual of the 5-form field-strength using Einstein-frame metric. Thus the field-strength associated with the gauge field $A^{(4)}$ as given in (8) is both self-dual and $\mathrm{SL}(2,\mathbb{Z})$ invariant.

(NS5, D0) solution.

$$\begin{aligned}
ds^2 &= H^{1/2}H'^{1/2}\left[-H^{-1}dx_0^2 + H'^{-1}\left(dx_1^2+\cdots+dx_5^2\right)+dr^2+r^2d\Omega_3^2\right]\\
e^{\phi_a} &= g_s H^{3/4}H'^{-1/4}\\
B^{(a)} &= 2Q_5\cos\varphi\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2\\
A^{(1)} &= \frac{\sin\varphi}{g_s}H^{-1}dx^0\\
A^{(3)} &= \frac{Q_5\sin\varphi\cos\varphi}{g_s}H^{-1}\sin^2\theta\cos\phi_1 dx^0\wedge d\theta\wedge d\phi_2\,.
\end{aligned}\tag{2.11}$$

Note here that the (NS5, D0), (NS5, D1) and (NS5, D2) solutions given above match precisely with the solutions in [11] apart from some unimportant constant term in RR gauge fields.

The ADM mass as well as the tension for these (NS5, Dp) bound states can be calculated by a generalization of the mass formula given in [23]. This has been done in [13] for (F, Dp) solutions. Here we use the same technique to obtain the tension of (NS5, Dp) bound states as

$$T_{(q,p)} = \frac{1}{g_s^2}\sqrt{p^2g_s^2+q^2}\,\frac{1}{(2\pi)^5\alpha'^3}\,. \tag{2.12}$$

Note that the tension is proportional to the charge Q_5 given in (2.3). Here $1/[(2\pi)^5\times\alpha'^3]$ is the fundamental tension of a 5-brane. For a single NS5-brane $q=1$, $p=0$, we recover the tension of an NS5-brane as $1/[g_s^2(2\pi)^5\alpha'^3]$. On the other hand for D5 brane $q=0$ and $p=1$, we recover the tension of a D5-brane as $1/[g_s(2\pi)^5\alpha'^3]$. For other Dp-branes with $p<5$, there are infinite number Dp-branes in the world-volume of NS5-brane lying along p spatial directions. 'p' in the expression[4] (2.12) denotes the charge of Dp-branes per $(2\pi)^{5-p}\alpha'^{(5-p)/2}$ of $(5-p)$-dimensional area of NS5-brane. So, for example, the tension of a D-string in (NS5, D1) bound state obtained from (2.12) is given by $1/g_sp((2\pi)^5\alpha'^3)(2\pi)^4\alpha'^2 = p/(g_s(2\pi\alpha'))$ as expected. It is clear from the expression (2.12) that when q, p are relatively prime integers (NS5, Dp) form non-threshold bound states.

3. SL$(2,\mathbb{Z})$ transformation and (NS5, D5, D3) bound state solution

We have obtained the type-IIB bound state (NS5, D3) in the previous section. Since type-IIB string theory is well-known to possess a non-perturbative quantum SL$(2,\mathbb{Z})$ symmetry, we will use it in this section on (NS5, D3) solution to construct (NS5, D5, D3) bound state solution. Note that (NS5, D1) solution also constructed in

[4]Although we have denoted the charge of a Dp-brane by the same integer 'p', they should not be confused and should be clear from the context.

the previous section belongs to type-IIB theory as well and we could use $\mathrm{SL}(2,\mathbb{Z})$ symmetry on this state to construct ((NS5, D5), (D1, F)) bound state, but this has already been done in [24]. So, (NS5, D5, D3) is the only new solution involving two D-branes in this case and we will construct it in this section following refs. [10, 25]. We here write the (NS5, D3) solution with $g_s = 1$, since this g_s has nothing to do with the asymptotic value of the dilaton in the final (NS5, D5, D3) bound state. (We will, however, restore the string coupling constant when we finally construct this bound state).

$$\begin{aligned}
ds^2 &= H^{1/2}H'^{1/2}\left[H^{-1}\left(-dx_0^2+dx_1^2+\cdots+dx_3^2\right)+H'^{-1}\left(dx_4^2+dx_5^2\right)+dr^2+r^2d\Omega_3^2\right]\\
e^{\phi_b} &= H'^{1/2}\\
B^{(b)} &= 2Q_5\cos\varphi\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2\\
\chi &= 0\\
A^{(2)} &= \frac{p}{q}(1-H'^{-1})dx^4\wedge dx^5\\
A^{(4)} &= -Q_5\sin\varphi(1+H'^{-1})\sin^2\theta\cos\phi_1 dx^4\wedge dx^5\wedge d\theta\wedge d\phi_2-\\
&\quad -\sin\varphi H^{-1}dx^0\wedge dx^1\wedge dx^2\wedge dx^3\,,
\end{aligned}\tag{3.1}$$

where

$$\begin{aligned}
H &= 1+\frac{Q_5}{r^2}\\
H' &= 1+\frac{\cos^2\varphi Q_5}{r^2}
\end{aligned}\tag{3.2}$$

with $Q_5=\sqrt{p^2+q^2}\alpha'$, $\cos\varphi = q/(\sqrt{p^2+q^2})$. Here p is the D3-brane charge and q is the NS5-brane charge. Also, the metric in the above is written in the string-frame and we have to write in the Einstein-frame since the Einstein-frame metric remains invariant under $\mathrm{SL}(2,\mathbb{Z})$ transformation. We will first make a classical $\mathrm{SL}(2,\mathbb{R})$ transformation on (13) and then impose the charge quantization to obtain the final (NS5, D5, D3) solution. The Einstein-frame metric has the form

$$\begin{aligned}
ds_E^2 &= e^{-\phi_b/2}ds^2 \qquad (3.3)\\
&= H^{1/2}H'^{1/4}\left[H^{-1}\left(-dx_0^2+dx_1^2+\cdots+dx_3^2\right)+H'^{-1}\left(dx_4^2+dx_5^2\right)+dr^2+r^2d\Omega_3^2\right].
\end{aligned}$$

If Λ denotes the global $\mathrm{SL}(2,\mathbb{R})$ transformation matrix then type-IIB fields transform under this transformation as follows:

$$\begin{aligned}
&g^E_{\mu\nu}\to g^E_{\mu\nu}\,, \qquad \lambda\to\frac{a\lambda+b}{c\lambda+d}\\
&\begin{pmatrix}B^{(b)}\\ A^{(2)}\end{pmatrix}\to(\Lambda^T)^{-1}\begin{pmatrix}B^{(b)}\\ A^{(2)}\end{pmatrix}, \qquad \begin{pmatrix}Q_1\\ Q_2\end{pmatrix}\to(\Lambda^T)^{-1}\begin{pmatrix}Q_1\\ Q_2\end{pmatrix}\\
&A^{(4)}\to A^{(4)}, \qquad Q_3\to Q_3\,,
\end{aligned}\tag{3.4}$$

where $\Lambda = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$, with $ad - bc = 1$. Also, 'T' here denotes the transpose of a matrix. $\lambda = \chi + ie^{-\phi}$ and Q_1 and Q_2 denote the charges of the NS5-brane and D5-brane respectively. Q_3 is the D3-brane charge which remains invariant under $\mathrm{SL}(2,\mathbb{Z})$. Note here that since the 5-brane charges are topological (magnetic), the charges transform in the same way as the gauge fields $B^{(b)}$ and $A^{(2)}$. If we assume the asymptotic value of the axion to be zero[5] and the asymptotic value of the dilaton to be ϕ_0, then the $\mathrm{SL}(2,\mathbb{R})$ matrix take the following form:

$$\Lambda = \begin{pmatrix} e^{-\phi_0/2}\cos\alpha & -e^{-\phi_0/2}\sin\alpha \\ e^{\phi_0/2}\sin\alpha & e^{\phi_0/2}\cos\alpha \end{pmatrix}, \tag{3.5}$$

where α is an unknown parameter to be determined from the charge quantization condition. For the initial (NS5, D3) configuration q was the charge of NS5-brane and D5-brane charge was zero, where q is an integer. We replace the initial charge of NS5-brane by an unknown number $\Delta^{1/2}$ (which is no longer an integer) and then impose the charge quantization after the $\mathrm{SL}(2,\mathbb{R})$ transformation. So,

$$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} e^{\phi_0/2}\cos\alpha & -e^{\phi_0/2}\sin\alpha \\ e^{-\phi_0/2}\sin\alpha & e^{-\phi_0/2}\cos\alpha \end{pmatrix} \begin{pmatrix} \Delta^{1/2} \\ 0 \end{pmatrix}. \tag{3.6}$$

Here (m, n) are integers and are associated with the charges of the NS5-brane and D5-brane respectively in the final configuration. From (3.7) we obtain,

$$\begin{aligned} \sin\alpha &= e^{\phi_0/2}\Delta^{-1/2} n \\ \cos\alpha &= e^{-\phi_0/2}\Delta^{-1/2} m\,. \end{aligned} \tag{3.7}$$

The above equation determines the value of Δ to be of the form

$$\Delta = \left(m^2 e^{-\phi_0} + n^2 e^{\phi_0}\right). \tag{3.8}$$

The $\mathrm{SL}(2,\mathbb{R})$ matrix Λ in (3.5) therefore takes the form

$$\Lambda = \frac{1}{\sqrt{m^2 e^{-\phi_0} + n^2 e^{\phi_0}}} \begin{pmatrix} e^{-\phi_0} m & -n \\ e^{\phi_0} n & m \end{pmatrix} \tag{3.9}$$

With this form of the $\mathrm{SL}(2,\mathbb{R})$ matrix, the dilaton and the axion for the (NS5, D5, D3) bound state are:

$$\begin{aligned} e^{\phi_b} &= e^{\phi_0} H'^{-1/2} H'' \\ \chi &= \frac{mn(1 - H')}{H''(m^2 + n^2 g_s^2)}, \end{aligned} \tag{3.10}$$

[5] This is taken for simplicity. One can construct the (NS5, D5, D3) solution with a non-zero asymptotic value of the axion (χ_0) by further making an $\mathrm{SL}(2,\mathbb{R})$ transformation on our solution given later in eqs.(25)–(28) with the $\mathrm{SL}(2,\mathbb{R})$ matrix $\left(\begin{smallmatrix} 1 & \chi_0 \\ 0 & 1 \end{smallmatrix}\right)$.

where H and H' are as given in (3.2) with q replaced by $\Delta^{1/2}$ (given in (3.8)) and H'' is given as

$$H'' = 1 + \frac{m^2 e^{-\phi_0} Q_5/(p^2 + m^2 e^{-\phi_0} + n^2 e^{\phi_0})}{r^2}\,. \tag{3.11}$$

The metric retains its form as given in (3.4), with the proper replacement of q as mentioned above. The four-form gauge field also remains invariant and takes the form as given in (3.1) whereas, the NSNS and RR two-form gauge fields transform according to eq.(3.4) and the final forms are

$$\begin{aligned} B^{(b)} &= \frac{2m}{\sqrt{m^2 e^{-\phi_0} + n^2 e^{\phi_0}}} Q_5 \cos\varphi \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2 - \\ &\quad - \frac{npe^{\phi_0}}{(m^2 e^{-\phi_0} + n^2 e^{\phi_0})}(1 - H'^{-1}) dx^4 \wedge dx^5 \\ A^{(2)} &= \frac{2n}{\sqrt{m^2 e^{-\phi_0} + n^2 e^{\phi_0}}} Q_5 \cos\varphi \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2 + \\ &\quad + \frac{mpe^{-\phi_0}}{(m^2 e^{-\phi_0} + n^2 e^{\phi_0})}(1 - H'^{-1}) dx^4 \wedge dx^5\,. \end{aligned} \tag{3.12}$$

So, we have constructed the (NS5, D5, D3) bound state with the Einstein metric in (3.4), the dilaton and axion as given in (3.10), the NSNS and RR two-form gauge fields given in (3.12) and the SL$(2, \mathbb{Z})$ invariant 4-form gauge field given in (3.1). Now we want to write this solution in the string frame such that the string-frame metric has the asymptotically Minkowskian form. Note here that in the (NS5, D5, D3) solution we have constructed the Einstein frame metric (eq. (3.4)) is asymptotically minkowskian. If we naively convert it into string frame by multiplying it with $e^{\phi_b/2}$, with e^{ϕ_b} given in (3.10), then the string frame metric does not become asymptotically Minkowskian. However, this can be achieved by scaling the coordinates as $(x_0, x_1, \cdots, x_5, r) \to e^{-\phi_0/4}(x_0, x_1, \cdots, x_5, r)$. We would also like to point out that in order to restore the correct g_s dependence, we have to replace p by $e^{\phi_0/2}p$ everywhere. The reason for this is that the D5-brane charge n and D3-brane charge p should have the same g_s factors multiplied since their masses and the charges have the same g_s dependence. So, with the above rescaling of the coordinates and the above replacement of the integer p, we can write down the (NS5, D5, D3) solution by the following string frame metric, dilaton, axion and other gauge fields,

$$\begin{aligned} ds^2 &= H^{1/2} H''^{1/2} \left[H^{-1}(-dx_0^2 + dx_1^2 + \cdots + dx_3^2) + H'^{-1}\left(dx_4^2 + dx_5^2\right) + dr^2 + r^2 d\Omega_3^2\right] \\ e^{\phi_b} &= g_s H'^{-1/2} H'' \\ \chi &= \frac{mn(1 - H')}{H''(m^2 + n^2 g_s^2)} = \frac{n}{m}(H''^{-1} - 1) \\ B^{(b)} &= \frac{2m}{\sqrt{m^2 + n^2 g_s^2}} Q_5 \cos\varphi \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2 - \frac{npg_s^2}{(m^2 + n^2 g_s^2)}(1 - H'^{-1}) dx^4 \wedge dx^5 \\ A^{(2)} &= \frac{2n}{\sqrt{m^2 + n^2 g_s^2}} Q_5 \cos\varphi \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2 + \frac{mp}{(m^2 + n^2 g_s^2)}(1 - H'^{-1}) dx^4 \wedge dx^5 \end{aligned}$$

$$
\begin{aligned}
A^{(4)} &= -\frac{Q_5}{g_s}\sin\varphi(1+H'^{-1})\sin^2\theta\cos\phi_1 dx^4\wedge dx^5\wedge d\theta\wedge d\phi_2 - \\
&\quad -\frac{\sin\varphi}{g_s}H^{-1}dx^0\wedge dx^1\wedge dx^2\wedge dx^3\,. \qquad (3.13)
\end{aligned}
$$

Here H and H' are as given in (3.2) and

$$
H'' = 1+\frac{m^2 Q_5/(m^2+(p^2+n^2)g_s^2)}{r^2} \qquad (3.14)
$$

$$
\cos\varphi = \frac{(m^2+n^2g_s^2)^{1/2}}{(m^2+(p^2+n^2)g_s^2)^{1/2}}\,. \qquad (3.15)
$$

The form of Q_5 is

$$
Q_5 = \sqrt{m^2+(p^2+n^2)g_s^2}\,\alpha'\,. \qquad (3.16)
$$

Thus eqs. (3.13)–(3.16) represent the (NS5, D5, D3) solution with m, n, p representing the integral charges for NS5-brane, D5-brane and D3-brane respectively. It can be easily checked that for $n = 0$, the above solution reduces to (NS5, D3) solution given in eq.(8) and for $p = 0$, it reduces to (NS5, D5) solution given in (2.1). Finally, for $m = 0$, this solution reduces to (D5, D3) solution constructed in ref. [26, 27].

4. T-duality and (NS5, D$(p+2)$, Dp) bound states

The (NS5, D5, D3) solution constructed in the previous section belongs to type-IIB theory, where NS5 and D5 branes are lying along the spatial x^1, x^2, x^3, x^4, and x^5 directions. D3 branes lie along x^1, x^2, x^3 directions. So, if we apply T-duality map from type-IIB theory to type-IIA theory given in eq. (2.4) along x^3 direction, then we will get the (NS5, D4, D2) bound state solution with D4-branes lying along x^1, x^2, x^4, x^5 coordinates and D2-branes lying along x^1, x^2 coordinates. A straightforward application of T-duality map produces the following (NS5, D4, D2) bound state solution:

(NS5, D4, D2) solution.

$$
\begin{aligned}
ds^2 &= H^{1/2}H''^{1/2}\Big[H^{-1}\left(-dx_0^2+dx_1^2+dx_3^2\right)+H''^{-1}dx_3^2+H'^{-1}\left(dx_4^2+dx_5^2\right)+dr^2+ \\
&\qquad\qquad + r^2 d\Omega_3^2\Big] \\
e^{\phi_a} &= g_s H^{1/4}H'^{-1/2}H''^{3/4} \\
B^{(a)} &= \frac{2m}{\sqrt{m^2+n^2g_s^2}}Q_5\cos\varphi\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2 - \frac{npg_s^2}{(m^2+n^2g_s^2)}(1-H'^{-1})dx^4\wedge dx^5 \\
A^{(1)} &= -\frac{n}{m}(H''^{-1}-1)dx^3 \\
A^{(3)} &= \frac{2n}{\sqrt{m^2+n^2g_s^2}}Q_5\cos\varphi\sin^2\theta\cos\phi_1 dx^3\wedge d\theta\wedge d\phi_2 + \\
&\quad +\frac{mp}{(m^2+n^2g_s^2)}(1-H'^{-1})dx^3\wedge dx^4\wedge dx^5 \; - \frac{\sin\varphi}{g_s}H^{-1}dx^0\wedge dx^1\wedge dx^2\,. \qquad (4.1)
\end{aligned}
$$

It can be easily checked from the above solution that for $p = 0$, it reduces to (NS5, D4) solution given in (2.6) and for $n = 0$, it reduces to (NS5, D2) solution given in (2.9). Also for $m = 0$, the above solution reduces to (D4, D2) solution (with additional isometries in x^3 direction) constructed in [26, 27]. Since the above bound state belongs to type-IIA theory, we apply the T-duality map from type-IIA to type-IIB fields in (2.7) along x^2 coordinate. Thus we obtain the (NS5, D3, D1) bound state where D3-branes lie along x^1, x^4, x^5 coordinates and D-strings lie along x^1 coordinate.

(NS5, D3, D1) solution.

$$
\begin{aligned}
ds^2 &= H^{1/2}H''^{1/2}\Big[H^{-1}\left(-dx_0^2 + dx_1^2\right) + H''^{-1}\left(dx_2^2 + dx_3^2\right) + H'^{-1}\left(dx_4^2 + dx_5^2\right) + \\
&\qquad + dr^2 + r^2 d\Omega_3^2\Big] \\
e^{\phi_b} &= g_s H^{1/2}H'^{-1/2}H''^{1/2} \\
B^{(b)} &= \frac{2m}{\sqrt{m^2+n^2g_s^2}} Q_5 \cos\varphi \sin^2\theta \cos\phi_1 d\theta \wedge d\phi_2 - \frac{npg_s^2}{(m^2+n^2g_s^2)}(1-H'^{-1})dx^4 \wedge dx^5 \\
\chi &= 0 \\
A^{(2)} &= -\frac{n}{m}(H''^{-1}-1)dx^2 \wedge dx^3 - \frac{\sin\varphi}{g_s}H^{-1}dx^0 \wedge dx^1 \\
A^{(4)} &= -\frac{n(H''^{-1}+1)}{\sqrt{m^2+n^2g_s^2}} Q_5 \cos\varphi \sin^2\theta \cos\phi_1 dx^2 \wedge dx^3 \wedge d\theta \wedge d\phi_2 - \\
&\quad - \frac{mp}{(m^2+n^2g_s^2)}(1-H'^{-1})dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5 + \\
&\quad + \frac{1}{2}\frac{n^2pg_s^2}{m(m^2+n^2g_s^2)}(H''^{-1}-1)(1-H'^{-1})dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5 \,.
\end{aligned} \tag{4.2}
$$

It can be checked from above that for $n = 0$, the solution reduces to (NS5, D1) solution given in eq. (2.10). However, for $p = 0$, it does not quite give the (NS5, D3) bound state in eq. (2.8). The reason for this is that, we have not included the other components of $A^{(4)}$ in eq. (4.2) obtained by the self-duality condition on the corresponding field strength. The field strength associated with $A^{(4)}$ is given below,

$$
\begin{aligned}
F^{(5)} &= -\frac{2n}{\sqrt{m^2+n^2g_s^2}} H''^{-1} Q_5 \cos\varphi \sin^2\theta \sin\phi_1 dx^2 \wedge dx^3 \wedge d\theta \wedge d\phi_1 \wedge d\phi_2 + \\
&\quad + \frac{mp}{m^2+n^2g_s^2} dH'^{-1} \wedge dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5 - \\
&\quad - \frac{n^2pg_s^2}{m(m^2+n^2g_s^2)}(H''^{-1}-1)dH'^{-1} \wedge dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5 + \\
&\quad + \frac{m\sin\varphi\cos\varphi}{g_s\sqrt{m^2+n^2g_s^2}} Q_5 \sin^2\theta \cos\phi_1 dH^{-1} \wedge dx^0 \wedge dx^1 \wedge d\theta \wedge d\phi_2 - \\
&\quad - \frac{m\sin\varphi\cos\varphi}{g_s\sqrt{m^2+n^2g_s^2}} Q_5 \sin^2\theta \sin\phi_1 H^{-1} dx^0 \wedge dx^1 \wedge d\theta \wedge d\phi_1 \wedge d\phi_2 -
\end{aligned}
$$

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$$
\begin{aligned}
&-\frac{1}{2}\frac{npg_s^2}{m^2+n^2g_s^2}\frac{\sin\varphi}{g_s}(1-H'^{-1})dH^{-1}\wedge dx^0\wedge dx^1\wedge dx^4\wedge dx^5 - \\
&-\frac{1}{2}\frac{npg_s^2}{m^2+n^2g_s^2}\frac{\sin\varphi}{g_s}H^{-1}dH'^{-1}\wedge dx^0\wedge dx^1\wedge dx^4\wedge dx^5\,. \qquad (4.3)
\end{aligned}
$$

Here, note that if we set $p=0$, all the terms in $F^{(5)}$ except the first term vanish. By simply taking the Hodge duality on this term correctly reproduces the form of $A^{(4)}$ in (NS5, D3) solution in eq. (2.8). We also note that by taking $m=0$, above solution again does not quite reduce to (D3, D1) solution (with additional isometries in x^2, x^3 directions) obtained in [26]–[28]. The reason again is that we have not included the terms obtained by Hodge duality in $A^{(4)}$. This is required because D3-branes are self-dual. However, for complicated bound state system involving D3, like the case we are considering, it is not clear how to write the Hodge dual terms in the gauge field since they produce non-local terms in general. But since we know the explicit form of the field strength we can verify that for the special case of $m=0$, the Hodge duality correctly reproduces the required terms in $A^{(4)}$. Thus we recover the (D3, D1) solution from above by setting $m=0$.

The (NS5, D3, D1) solution constructed above belongs to type-IIB theory. So, we use the T-duality map given in (2.4) along x^1-coordinate to obtain (NS5, D2, D0) solution. Here NS5-branes lie along x^1, $x^2,\cdots,x^5$ directions and D2-branes along x^4, x^5-directions.

(NS5, D2, D0) solution.

$$
\begin{aligned}
ds^2 &= H^{1/2}H''^{1/2}\Big[-H^{-1}dx_0^2+H''^{-1}\left(dx_1^2+dx_2^2+dx_3^2\right)+H'^{-1}\left(dx_4^2+dx_5^2\right)+\\
&\qquad\qquad + dr^2+r^2d\Omega_3^2\Big]\\
e^{\phi_a} &= g_sH^{3/4}H'^{-1/2}H''^{1/4}\\
B^{(a)} &= \frac{2m}{\sqrt{m^2+n^2g_s^2}}Q_5\cos\varphi\sin^2\theta\cos\phi_1 d\theta\wedge d\phi_2 - \frac{npg_s^2}{(m^2+n^2g_s^2)}(1-H'^{-1})dx^4\wedge dx^5\\
A^{(1)} &= \frac{\sin\varphi}{g_s}H^{-1}dx^0\\
A^{(3)} &= -\frac{n}{m}(H''^{-1}-1)dx^1\wedge dx^2\wedge dx^3+\frac{m\sin\varphi\cos\varphi}{g_s\sqrt{m^2+n^2g_s^2}}Q_5H^{-1}\sin^2\theta\cos\phi_1\,\times\\
&\qquad \times dx^0\wedge d\theta\wedge d\phi_2-\frac{1}{2}\frac{npg_s}{(m^2+n^2g_s^2)}\sin\varphi H^{-1}(1-H'^{-1})dx^0\wedge dx^4\wedge dx^5\,. \qquad (4.4)
\end{aligned}
$$

Again for $n=0$, we recover the (NS5, D0) solution given in eq.(2.11). Similarly, by setting $m=0$, we recover the (D2, D0) bound state solution (with additional isometries in x^1, x^2, x^3 directions) obtained in [26]–[28]. However, for $p=0$, we can only recover (NS5, D2) solution given in eq. (2.9) if we include the Hodge dual terms in $A^{(4)}$ of (NS5, D3, D1) solution in eq. (4.2) for this special case as mentioned before.

The ADM mass and the tension of (NS5, D$(p+2)$, Dp) bound states can be obtained by a further generalization of the mass formula given in [13]. We find that it is given by,

$$T_{(m,n,p)} = \frac{1}{g_s^2}\sqrt{(p^2+n^2)g_s^2+m^2}\,\frac{1}{(2\pi)^5\alpha'^3}\,. \tag{4.5}$$

Again it is proportional to the charge Q_5 given in (3.16). It can be easily verified from (4.5) that when any two of the integers m, n, p are relatively prime the (NS5, D$(p+2)$, Dp) form non-threshold bound states.

We would like to point out that one can make further SL$(2,\mathbb{Z})$ transformation on (NS5, D3, D1) to construct the most general bound state involving NS5-branes and lower Dp-branes of the form (NS5, D5, D3, D1, F) of type-IIB string theory. Furthermore, if we take T-duality on this state along x^1-direction, we obtain type-IIA bound state of the form (NS5, D4, D2, D0, W), where 'W' denotes the waves. However, it is not clear how to obtain a similar state involving F-string in this case as given in [12].

5. ODp-limit

It has been shown in [21] that there exist a series of new six-dimensional theories which are nothing but the decoupled theories of NS5-branes in the presence of a critical RR $(p+1)$-form gauge field whose excitations include light open Dp-branes known as ODp theories. The dual supergravity solutions of these theories are the particular decoupling limit of the (NS5, Dp) bound state configurations constructed in section 2. Although these supergravity solutions have already been given in [11], we here briefly discuss these solutions since our solutions differ from those in [11]. The ODp-decoupling limit is defined as follows:

$$\cos\varphi = \epsilon \to 0 \tag{5.1}$$

keeping the following quantities fixed

$$\alpha'_{\text{eff}} = \frac{\alpha'}{\epsilon}\,, \quad u = \frac{r}{\epsilon\alpha'_{\text{eff}}}\,, \quad G^2_{o(p)} = \epsilon^{(p-3)/2} g_s\,, \quad Q_5 = \alpha'_{\text{eff}} q\,. \tag{5.2}$$

The harmonic functions take the forms

$$H = \frac{1}{\epsilon^2 a^2 u^2}\,, \qquad H' = \frac{h}{a^2 u^2}\,, \tag{5.3}$$

where $h = 1 + a^2u^2$, with $a^2 = \alpha'_{\text{eff}}/q$. The metric, dilaton and the NSNS 2-form are:

$$\begin{aligned} ds^2 &= \alpha' h^{1/2}\left[-d\tilde{x}_0^2 + \sum_{i=1}^{p} d\tilde{x}_i^2 + h^{-1}\sum_{j=p+1}^{5} d\tilde{x}_j^2 + \frac{q}{u^2}\left(du^2 + u^2 d\Omega_3^2\right)\right] \\ e^{\phi_{(a,b)}} &= G^2_{o(p)}\frac{h^{(p-1)/4}}{au} \\ B^{(a,b)}_{\theta\phi_2} &= 2\alpha' q \sin^2\theta\cos\phi_1\,, \end{aligned} \tag{5.4}$$

where $\tilde{x}_{0,\cdots,p} = 1/(\sqrt{\alpha'_{\rm eff}})x_{0,\cdots,p}$, $\tilde{x}_{p+1,\cdots,5} = \sqrt{\alpha'_{\rm eff}}/\alpha' x_{p+1,\cdots,5}$ = fixed. The quantization condition is

$$\frac{p}{q} = \frac{\tan\varphi}{g_s} = \frac{1}{G^2_{o(p)}\epsilon^{(5-p)/2}} \tag{5.5}$$

The RR gauge fields take the following forms for different values of p,

$$\begin{aligned}
p = 0\,, &\quad A^{(1)}_0 = \frac{\alpha'^{1/2}}{G^2_{o(0)}}(au)^2\,, \quad A^{(3)}_{0\theta\phi_2} = \frac{\alpha'^{3/2}q}{G^2_{o(0)}}(au)^2\sin^2\theta\cos\phi_1\\
p = 1\,, &\quad \chi = 0, \quad A^{(2)}_{01} = -\frac{\alpha'}{G^2_{o(1)}}(au)^2\,, \quad A^{(4)}_{2345} = \frac{\alpha'^2}{G^2_{o(1)}}\frac{(au)^2}{h}\\
p = 2\,, &\quad A^{(1)} = 0, \quad A^{(3)}_{012} = -\frac{\alpha'^{3/2}}{G^2_{o(2)}}(au)^2\,, \quad A^{(3)}_{345} = -\frac{\alpha'^{3/2}}{G^2_{o(2)}}\frac{(au)^2}{h}\\
p = 3\,, &\quad \chi = 0, \quad A^{(2)}_{45} = -\frac{\alpha'}{G^2_{o(3)}}\frac{(au)^2}{h}\,, \quad A^{(4)}_{45\theta\phi_2} = \frac{\alpha'^2 q}{G^2_{o(3)}}(1+h^{-1})\sin^2\theta\cos\phi_1\,,\\
&\quad A^{(4)}_{0123} = -\frac{\alpha'^2}{G^2_{o(3)}}(au)^2\\
p = 4, &\quad A^{(1)}_5 = -\frac{\alpha'^{1/2}}{G^2_{o(4)}}\frac{(au)^2}{h}, \quad A^{(3)}_{5\theta\phi_2} = -\frac{2q\alpha'^{3/2}}{G^2_{o(4)}}\sin^2\theta\cos\phi_1\\
p = 5\,, &\quad \chi = -\frac{1}{G^2_{o(5)}}\frac{1}{h}, \quad A^{(2)}_{\theta\phi_2} = -2p\alpha'\sin^2\theta\cos\phi_1
\end{aligned} \tag{5.6}$$

Note that in writing down the above solutions we have thrown away some constant pieces in the gauge fields $A^{(4)}_{2345}$, $A^{(3)}_{345}$, $A^{(2)}_{45}$ and $A^{(1)}_5$ in $p = 1, 2, 3, 4$ respectively. For $p = 3$ we have added the dual of the gauge field $A^{(4)}_{0123}$ and this is crucial to have the correct form of (NS5, D4) and (NS5, D5) solutions written in (2.1) and (2.6) if we start from (NS5, D1) solution and apply T-duality to obtain them as done in [11]. However, the metric and the dilaton which essentially determine the behavior of the theory at various regimes of the energy parameter u have the same form as in [11] and therefore, the conclusions remain the same. We will here briefly discuss the ODp-limits for different values of p.

It has been noted in refs.[29, 30] that ODp-limit (at least for $p = 1, 2$), is different from the little string theory limit $g_s \to 0$ and α' =fixed. However, it has been clarified later in [11] (also in [30]), that for $p \leq 2$, one can take a decoupling limit different from (5.2) and resembles the little string theory limit as follows:[6]

$$\cos\varphi = \epsilon \to 0, \quad g_s = \epsilon^{(3-p)/2}G^2_{o(p)} \to 0, \quad u = \frac{r}{\sqrt{\alpha'}\sqrt{\epsilon}G^{2/(3-p)}_{o(p)}} = \text{fixed}, \quad \alpha' = \text{fixed}. \tag{5.7}$$

[6]For $p = 1$ and $p = 2$, these limits have been referred to in [30] as the open brane little string theory (OBLST) limits. The solutions (37)–(39) in these cases are the supergravity dual descriptions of (1, 1) OBLST and (0, 2) OBLST respectively. We would like to thank T. Harmark for pointing this out to us.

The supergravity solution reduces precisely to the same form as in ODp given in eqs. (5.4)–(5.6) with the coordinate scaling

$$\tilde{x}_{0,\cdots,p} = \sqrt{\frac{\epsilon}{\alpha'}} x_{0,\cdots,p} = \text{fixed}\,, \qquad \tilde{x}_{p+1,\cdots,5} = \frac{1}{\sqrt{\alpha'\epsilon}} x_{p+1,\cdots,5} = \text{fixed}\,, \tag{5.8}$$

but now $\alpha' = $ fixed (as opposed to ODp-limit where $\alpha' \to 0$) and $a^2 = G_{o(p)}^{4/(3-p)}/q$. Indeed, we note from eq. (5.4) that for $au \ll 1$, we recover (5+1)-dimensional Lorentz invariance and the supergravity solutions in this case reduce to those of little string theories given in [31, 32].

Now coming back to ODp theories, we note that the curvature of the metric in eq. (5.4) measured in units of α' i.e. $\alpha'\mathcal{R} \sim 1/q$ and so, supergravity solution can be trusted for large enough q (no. of NS5-branes). When $au \ll 1$, i.e. in the IR region the supergravity solution is valid only if $au \gg G_{o(p)}^2$, where $e^{\phi_{(a,b)}}$ remains small. In this case $G_{o(p)}^2 \ll 1$. However, this condition is not satisfied in the extreme IR region, where $e^{\phi_{(a,b)}}$ is large. In that case, we have to either go to the S-dual frame for type-IIB theory or lift the solution to M-theory for type-IIA theory to have valid supergravity description. The ODp-theories in this case flow to (5+1)-dimensional SYM theory for $p =$ odd and for $p =$ even, they flow to (0, 2) superconformal field theory.

For $au \gg 1$, i.e. in the UV region, the supergravity solution is valid if $(au)^{(3-p)/2} \gg G_{o(p)}^2$. So, for $p = 0, 1, 2$ we have $au \gg G_{o(0)}^{4/3}$, $au \gg G_{o(1)}^2$ and $au \gg G_{o(2)}^4$ respectively. In the extreme UV region, these conditions are always satisfied and we have good supergravity description. The form of the metric in these cases reduce to those of ordinary Dp-branes with additional isometries in $(5 - p)$ directions. The reason for this can be understood from the quantization relation eq. (5.5), where we note that the Dp-branes dominate over NS5-branes.

For $p = 3$, the string coupling $e^{\phi_b} = G_{o(3)}^2 =$ fixed. So, when $G_{o(3)}^2 \ll 1$, the metric reduces to that of ordinary D3-branes with additional isometries in 4, 5 directions. This can also be understood from the quantization relation (5.5). When $G_{o(3)}^2 \gg 1$, we have to go to the S-dual frame and (NS5, D3) solution becomes (D5, D3) system and thus the strongly coupled OD3 theory become equivalent to six-dimensional NCYM with noncommutativity parameter $\theta = 2\pi\alpha'_{\text{eff}} G_{o(3)}^2$ and the coupling $g_{YM}^2 = (2\pi)^3 \alpha'_{\text{eff}}$.

For $p = 4$, the string coupling $e^{\phi_a} = G_{o(4)}^2 (au)^{1/2}$. So, the supergravity solution is valid if $au \ll G_{o(4)}^{-4}$. In the extreme UV region this condition is not satisfied and therefore, the supergravity description would have to be given by lifting the solution to M-theory. The supergravity solution is then given by the two intersecting M5-branes along 1,2,3,4 directions.

For $p = 5$, the string coupling $e^{\phi_b} = G_{o(5)}^2 au$. So, in order to have valid supergravity description $au \ll G_{o(5)}^{-2}$. This again is not satisfied if we are in the extreme UV region. In this case, we have to go to the S-dual frame. Note from (5.6) that in UV, $\chi \to 0$ (This is because in our solution of (NS5, D5) given in eq. (2.1), the asymptotic value of χ vanishes) and therefore, the string coupling in the S-dual frame is given as $e^{\phi'_b} = (G^2_{o(5)} au)^{-1}$ and remains small. The metric $ds'^2 = e^{-\phi_b} g_s ds^2$ and the dilaton as given above then reduces to those of little string theory with $g'_s \to 0$ and $\alpha'_{\rm eff}$ = fixed. We note from eq. (5.5) that $G^2_{o(5)} = q/p$ is quantized and the axion in (5.6) reduces to a rational number in IR, whereas it vanishes in the UV. Here, our conclusion differs from ref. [11] and this is because we have taken the asymptotic value of the axion to be zero, whereas in ref. [11], it was taken to be constant.

Note added

After submission of this paper to the net we were informed by M. Cederwall, U. Gran, M. Nielsen and B.E.W. Nilsson that some of the solutions (for type IIB) constructed in this paper were also considered in [33] from a different approach.

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References

[1] A. Strominger, *Heterotic solitons*, *Nucl. Phys.* **B 343** (1990) 167.

[2] C.G. Callan Jr., J.A. Harvey and A. Strominger, *World sheet approach to heterotic instantons and solitons*, *Nucl. Phys.* **B 359** (1991) 611.

[3] S.-J. Rey, *The confining phase of superstrings and axionic strings*, *Phys. Rev.* **D 43** (1991) 526.

[4] M.J. Duff and J.X. Lu, *Elementary five-brane solutions of d=10 supergravity*, *Nucl. Phys.* **B 354** (1991) 141.

[5] M.J. Duff, R.R. Khuri and J.X. Lu, *String solitons*, *Phys. Rep.* **259** (1995) 213 [hep-th/9412184].

[6] A. Dabholkar and J.A. Harvey, *Nonrenormalization of the superstring tension*, *Phys. Rev. Lett.* **63** (1989) 478.

[7] A. Dabholkar, G. Gibbons, J.A. Harvey and F.R. Ruiz, *Superstrings and solitons*, *Nucl. Phys.* **B 340** (1990) 33.

JHEP02(2001)026

[8] G.T. Horowitz and A. Strominger, *Black strings and p-branes*, *Nucl. Phys.* **B 360** (1991) 197.

[9] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[10] J.X. Lu and S. Roy, *An* SL*(2,Z) multiplet of type IIB super five-branes*, *Phys. Lett.* **B 428** (1998) 289 [hep-th/9802080].

[11] M. Alishahiha, Y. Oz and J. Russo, *Supergravity and light-like non-commutativity*, *J. High Energy Phys.* **09** (2000) 002.

[12] M. Alishahiha and Y. Oz, *Supergravity and 'new' six-dimensional gauge theories*, *Phys. Lett.* **B 495** (2000) 418 [hep-th/0008172].

[13] J.X. Lu and S. Roy, *Non-threshold (f,d p) bound states*, *Nucl. Phys.* **B 560** (1999) 181 [hep-th/9904129].

[14] M. Berkooz, M. Rozali and N. Seiberg, *Matrix description of M theory on T^4 and T^5*, *Phys. Lett.* **B 408** (1997) 105 [hep-th/9704089].

[15] N. Seiberg, *New theories in six dimensions and matrix description of M- theory on T^5 and $T^{5/z(2)}$*, *Phys. Lett.* **B 408** (1997) 98 [hep-th/9705221].

[16] R. Dijkgraaf, E. Verlinde and H. Verlinde, *BPS quantization of the five-brane*, *Nucl. Phys.* **B 486** (1997) 89 [hep-th/9604055].

[17] R. Dijkgraaf, E. Verlinde and H. Verlinde, 5*d black holes and matrix strings*, *Nucl. Phys.* **B 506** (1997) 121 [hep-th/9704018].

[18] A. Losev, G. Moore and S.L. Shatashvili, *M&mś*, *Nucl. Phys.* **B 522** (1998) 105 [hep-th/9707250].

[19] O. Aharony, *A brief review of 'little string theories'*, *Class. and Quant. Grav.* **17** (2000) 929 [hep-th/9911147].

[20] M. Alishahiha, Y. Oz and M.M. Sheikh-Jabbari, *Supergravity and large-N noncommutative field theories*, *J. High Energy Phys.* **11** (1999) 007 [hep-th/9909215].

[21] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, *J. High Energy Phys.* **08** (2000) 008.

[22] E. Bergshoeff, C. Hull and T. Ortin, *Duality in the type II superstring effective action*, *Nucl. Phys.* **B 451** (1995) 547 [hep-th/9504081].

[23] J.X. Lu, *Adm masses for black strings and p-branes*, *Phys. Lett.* **B 313** (1993) 29 [hep-th/9304159].

[24] J.X. Lu and S. Roy, $(f, d5)$ *bound state,* SL*(2,Z) invariance and the descendant states in type IIB/a string theory*, *Phys. Rev.* **D 60** (1999) 126002 [hep-th/9905056].

[25] J.X. Lu and S. Roy, *(m, n)-string-like Dp-brane bound states*, *J. High Energy Phys.* **08** (1999) 002 [hep-th/9904112].

[26] J.C. Breckenridge, G. Michaud and R.C. Myers, *More D-brane bound states*, *Phys. Rev.* **D 55** (1997) 6438 [hep-th/9611174].

[27] M.S. Costa and G. Papadopoulos, *Superstring dualities and p-brane bound states*, *Nucl. Phys.* **B 510** (1998) 217 [hep-th/9612204].

[28] J.G. Russo and A.A. Tseytlin, *Waves, boosted branes and BPS states in M-theory*, *Nucl. Phys.* **B 490** (1997) 121 [hep-th/9611047].

[29] M. Alishahiha, *On type II NS5-branes in the presence of an RR field*, *Phys. Lett.* **B 486** (2000) 194.

[30] T. Harmark, *Open branes in space-time non-commutative little string theory*, *Nucl. Phys.* **B 593** (2001) 76 [hep-th/0007147].

[31] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, *Supergravity and the large-N limit of theories with sixteen supercharges*, *Phys. Rev.* **D 58** (1998) 046004 [hep-th/9802042].

[32] O. Aharony, M. Berkooz, D. Kutasov and N. Seiberg, *Linear dilatons, ns5-branes and holography*, *J. High Energy Phys.* **10** (1998) 004 [hep-th/9808149].

[33] M. Cederwall, U. Gran, M. Nielsen and B. E. W. Nilsson, *J. High Energy Phys.* **01** (2000) 037.

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